On the Shallow Decay of Some GRB Afterglows

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Abstract. Half of the radio afterglows for which there is a good temporal coverage exhibit after 10 days from the burst a decay which is shallower than at optical frequencies, contrary to what is expected within the simplest form of the standard model of relativistic fireballs or jets. We investigate possible ways to decouple the radio and optical decays. First, the radio and optical emissions are assumed to arise from the same electron population and we allow for either a time-varying slope of the power-law distribution of electron energy or for time-varying microphysical parameters. Then we consider two scenarios where the radio and optical emissions arise in distinct parts of the GRB outflow, either because the outflow has an angular structure or because there is a long-lived reverse shock. We find that the only the last scenario is compatible with the observations.

The anomalous radio afterglows. The radio emission of all well-observed GRB afterglows decays after about day 10. This is consistent with what is expected in the standard fireball model for GRB afterglows. In this model, the characteristic synchrotron frequency ν_i at which electrons with the typical post-shock energy $e_i = \varepsilon_i m_p c^2 \Gamma$ (Γ being the fireball Lorentz factor) radiate is

$$\nu_i \sim 30 \left(\frac{\mathscr{E}}{10^{53}\,\mathrm{ergs}}\right)^{1/2} \left(\frac{\varepsilon_i}{0.03}\right)^2 \left(\frac{\varepsilon_B}{10^{-3}}\right)^{1/2} \left(\frac{t}{10\,\mathrm{d}}\right)^{-3/2} \mathrm{GHz} \tag{1}$$

where $\mathscr{E}$ is the fireball's kinetic energy per solid angle and the parameters ε_i and ε_B quantify the fraction of the post-shock energy imparted to electrons[1] and to the magnetic field. If the outflow is collimated, the evolution of ν_i becomes faster, $\nu_i \propto t^{-2}$, after the "jet-break" time when the jet start expanding laterally.

Equation (1) indicates that, for reasonable afterglow parameters, the injection frequency ν_i is expected to cross the radio domain around 10 days, after which the radio afterglow should decay as $F_r \propto t^{-\alpha_r}$ with $\alpha_r = (3p-1)/4$ for a fireball interacting with a wind medium, $\alpha_r = (3p-3)/4$ for a fireball decelerated by a uniform medium, and $\alpha_r = p$ for jet spreading laterally. In the standard afterglow model, the optical light-curve decay is expected to be the same, apart from a difference $|\alpha_o - \alpha_r| = 1/4$ arising when the cooling frequency (ν_c) is below the optical domain.

For five radio afterglows (970508, 980329, 980703, 000418, 021004), the radio and optical decay indices are consistent with this basic "prediction" of the fireball model, but it is not so for the other five well-monitored radio afterglows (991208, 991216, 000301, 000926, 010222), which are shown in Figure 1 . For these cases, it is natural

[1] For a power-law distribution of electron energies $dN/de \propto e^{-p}$ at $e > e_i$, the total electron energy is $[(p-1)/(p-2)] \times \varepsilon_i$ for $p > 2$.

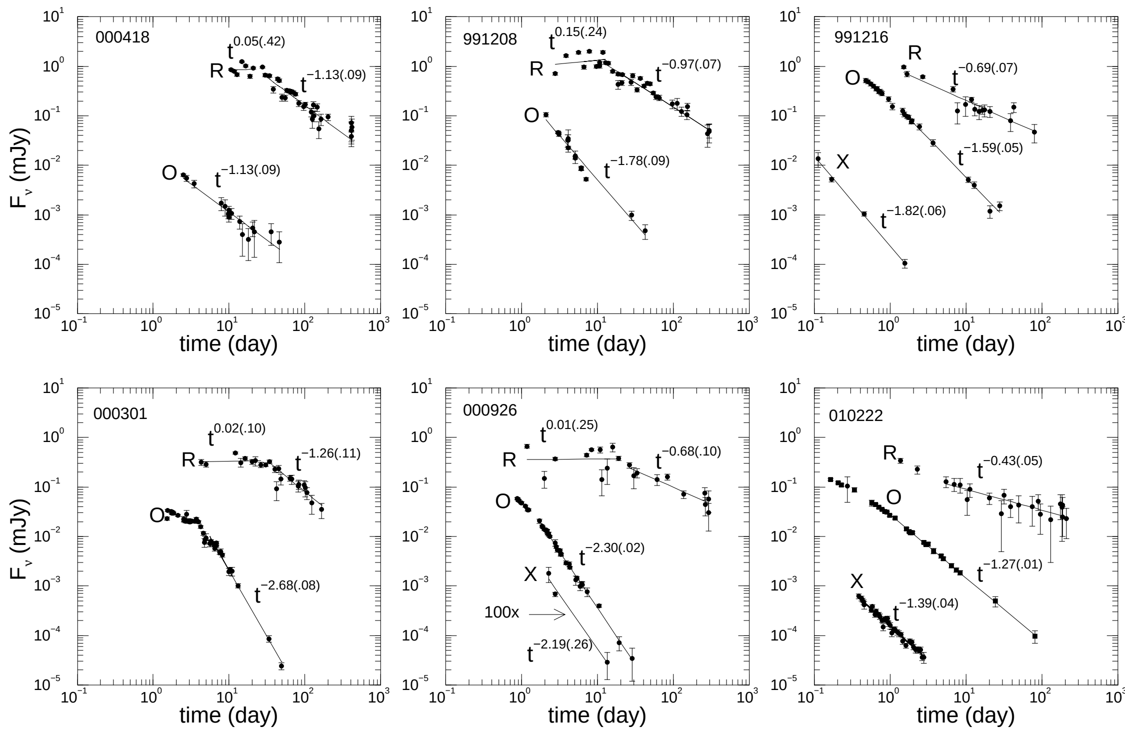

FIGURE 1. Radio (8 GHz), Optical (5×10^{14} Hz), and X-ray (5 keV) light-curves for five anomalous GRB afterglows whose radio light-curves decay slower than in the optical and for the afterglow 000418, for which the decays seen in these domains have the same indices, as expected in the simplest version of the standard fireball/jet afterglow model. Power-law fits are indicated for each frequency range, errors are given in parentheses.

to investigate first if the difference between the radio and optical decay indices could be caused by that the injection frequency ν_i remains above the radio domain (of typical frequency $\nu_r \sim 10$ GHz) until the last radio measurements[2] , usually around 100 days after the burst. According to equation (1), this requires electron and/or magnetic field parameters close to equipartition ($\varepsilon_i, \varepsilon_B \gtrsim 0.1$), particularly if the jet-break time occurs at around 1 day, as is the case for the afterglows 991216, 000301, 000926 and 010222 (see the break exhibited by their optical light-curves).

For $\nu_r < \nu_i$ there are three relevant cases which yield a decaying radio emission. If the GRB ejecta is spherical (or a sufficiently wide jet), the radio light-curve decays only if the circumburst medium is a wind (r^{-2} density profile) and if $\nu_r < \nu_c < \nu_i$, in which case $\alpha_r = 2/3$, as observed for the afterglows 991216 and 000926. If the GRB ejecta is collimated, a decaying radio light-curve can be obtained either for $\nu_i < \nu_c$ or for $\nu_c < \nu_i$. In the former case $\alpha_r = 1/3$, close to the value observed for the afterglow 010222, while the latter case yields $\alpha_r = 1$, as seen in the afterglow 991208. Despite this

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[2] That radio afterglows decay after 10 days indicate that the self-absorption frequency is below the radio domain, thus it cannot account for this difference

nice agreement between the expected and observed radio decay indices for $\nu_r < \nu_i$, none of the above cases actually work well. The first and third case require dense external media (homogeneous or wind-like) to maintain $\nu_c < \nu_i$ until about 100 days, leading to self-absorbed sources at radio frequencies even at day 10, which is inconsistent with the decaying radio light-curves, and to a synchrotron flux at ν_i above 100 mJy at 100 days, which is 1,000 times larger than the radio flux observed at that time. The second case leads to very tenuous external media, which are rather inconsistent with those expected for a massive star GRB progenitor.

Thus we have to investigate departures from the standard afterglow model in its simplest form that could account for the shallow radio decays observed for the above five anomalous afterglows. Since the $\nu_r < \nu_i$ case cannot explain all the properties of the radio afterglow emission, we will consider that the injection frequency ν_i is below the radio domain when the radio decay is observed. In fact, the passage of ν_i through this domain is the most natural cause for the onset of the radio decay seen at about 10 days in the afterglows 991208, 000301, 000926 and 010222, as well as in other afterglows whose radio and optical decay indices are consistent with the fireball model expectations.

These departures fall in two categories. In the first, we assume that the radio and optical emissions arise from the same region of the GRB fireball. If the same electron population gives both the radio and optical afterglows, then different decay indices can be obtained either if the slope p of the electron energy distribution is time-varying or if there is a spectral break between these two domains. In the second category, we assume that the radio and optical emissions arise from different fireball regions, as could be the case with a structured fireball or a long-lived reverse shock. We note that, in the scenarios that we describe below, involving time-varying afterglow parameters, the break seen in the optical light-curve of most afterglows shown in Figure 1 may arise from a rapid variation of the parameters under investigation and not necessarily from the tight collimation of the GRB outflow.

Electron distribution with time-varying slope. We consider that, at all times of interest (1-100 days after the burst), the electron distribution injected by the forward shock is a power-law of exponent $-p$ that varies in time[3] . It is evident that, in this case, the radio and optical light-curves cannot both be power-laws in time. Given that optical measurements have smaller uncertainties, we can determine the evolution of the electron index p that yields a power-law optical light-curve and test the consequences of that evolution on the afterglow emission at other frequencies.

It can be shown that, in order to obtain $\alpha_r < \alpha_o$, the index p must increase in time, asymptotically approaching the value that the observed α_r required if the index p were constant in time. Thus the optical afterglow spectrum should soften in time. For such a behavior of p, the radio light-curve should steepen in time, while the X-ray light-curve should decay faster that in the optical and should flatten in time. In general, these features are not quantitatively consistent with the multiwavelength observations of the

[3] An alternative scenario for the $\alpha_o - \alpha_r$ index difference of the anomalous afterglows is an electron distribution that is not a power-law and whose shape does not change in time. This scenario is largely unconstrainable with current observations.

anomalous afterglows: the optical spectrum of 991208 appears to harden in time while the decays of the X-ray light-curve of 991216 (during day 1) and of the radio light-curves of 000926 and 010222 (at 100 days) are much shallower than expected. Only for 000301 the radio light-curve and optical spectrum are consistent with the consequences of an evolving electron distribution index.[4]

Variable electron and magnetic field parameters. As the self-absorption and injection break frequencies must be below the radio domain when the radio light-curves decay, the only remaining break that could decouple the radio and optical decay indices must be the cooling frequency ν_c. However its evolution must be much faster than that for a constant magnetic parameter ε_B, to account for the magnitude of the observed $\alpha_o - \alpha_r$. Since we want, in fact, to explain not just the difference $\alpha_o - \alpha_r$, but the observed values of α_r and α_o, we must also allow for the electron energy parameter ε_i to be time-varying. We shall also consider that the fireball's kinetic energy per solid angle $\mathscr{E}$ (or the kinetic energy for a jet) is time-varying, either because of radiative losses or an energy injection in the fireball through some less relativistic ejecta which catch-up with the leading edge of the ejecta (which is decelerated by the interaction with the circumburst medium).

Because the quantities pertaining to the fireball dynamics are power-laws in the observer time t, it is then natural to restrict our attention to time-varying ε_i, ε_B, and $\mathscr{E}$ that evolve as power-laws with t: $\mathscr{E} \propto t^e$, $\varepsilon_i \propto t^i$, $\varepsilon_B \propto t^b$. Radiative losses yield $e \geq -3/7$ for a fireball interacting with a homogeneous medium, $e \geq -1/3$ if the medium is wind-like, and $e \geq -3/5$ for a spreading jet and any type of external medium. Using standard equations for the afterglows spectral characteristics, we can calculate the decay indices for the radio and optical light-curves as functions of the parameters e, i and b and use the observed α_r and α_o to constraint them.

In this way it can be shown that, if there is no energy injection ($e \leq 0$), then ε_i must decrease in time while ε_B must increase so fast that the magnetic field strength $B \propto \varepsilon_B^{1/2}$ is constant or increases in time. This rather extreme requirement is somewhat alleviated if the external medium density increases with radius, as such a density profile leads to a faster evolution of the cooling frequency. If there is an energy injection, leading to $e \geq 0$, then there are solutions with constant ε_B, however the peak flux and injection frequency evolutions implied by this scenario are in strong conflict with those inferred from the radio emission of the afterglow 991208. These conclusions apply to both a spherical fireball and a collimated outflow.

Structured outflow. If the outflow has a non-uniform angular distribution of the kinetic energy per solid angle $\mathscr{E}$, it is possible that the optical emission arises predominantly from a core of higher $\mathscr{E}$ while the radio afterglow is emitted by a surrounding envelope of lower $\mathscr{E}$. This possibility is suggested by the dependence of the injection frequency ν_i on $\mathscr{E}$ (eq. [1]). For simplicity and maximal effect of the outflow structure, we consider that both the core and envelope have an uniform distribution of $\mathscr{E}$. In this scenario, the break exhibited by the afterglow optical light-curve is caused by the collimation of the core, while the onset of the radio decay is due to the passage through the

[4] There are no X-ray measurements for the afterglow 000301 to further test this scenario

radio domain of the ν_i for the envelope emission. To explain the observed α_r and α_o, we must allow for different electron indices p in the outflow core and envelope.

The test that this scenario must pass is that the radio and optical emissions are decoupled. More specifically, the softening emission from the optical core must not overshine in the radio the emission from the envelope and the emission from the envelope must not be brighter in the optical than the faster decaying emission from the core. The first condition is equivalent to a lower limit on the ν_i frequency for the optical core at the time when the optical light-curve break is observed (which is the time when the core edge becomes visible to the observer). With the aid of equation (1), this leads to a lower limit on the injection frequency for the radio envelope at the time when the radio light-curve begins to decay. For the parameters of the radio and optical emission of the five anomalous afterglows, the latter lower limit falls invariably above 10 GHz, contrary to what is implied by the observed onset of the radio decay. The second condition leads to an upper limit on the cooling frequency for the envelope emission, which implies very dense external media and a self-absorbed radio emission, inconsistent with the observed radio decay.

Reverse shock. If there is a continuous inflow of slower ejecta from the GRB progenitor into the leading edge of the GRB fireball, then the emission from the long-lived reverse shock crossing the incoming ejecta could dominate the radio afterglow emission, while the optical afterglow arises as usually from the forward shock. Given that a collimated outflow undergoing lateral spreading and delayed energy injection loses angular uniformity, we restrict our attention to a spherical outflow, which maintains its isotropy during the injection.

For simplicity, we parameterize the distribution of ejecta mass with Lorentz factor as a power-law, $d\mathscr{M}_i/d\Gamma \propto \Gamma^{-(q+1)}$. The fireball energy can also vary in time (e.g. $\mathscr{E} \propto t^e$), as described for a structured outflow[5] . Just as for the scenario involving variable microphysical parameters, one can calculate the radio and optical decay indices as function of the parameters q and e, and then determine these parameters with the aid of observations. We find that the anomalous radio afterglows require $q > 3$ for a homogeneous medium, $q > 4$ for a wind medium, and $e < 1/3$, indicating that the incoming ejecta can carry at most the same energy as the initial outflow energy.

The reverse-forward shock scenario must also pass the test discussed above for a structured outflow, leading to a lower limit on the forward shock ν_i frequency and an upper limit on the reverse shock cooling frequency. Two other constraints can be obtained by requiring that the forward shock yields the flux normalization seen in the optical and that the ν_i frequency for the reverse shock crosses the radio domain when the onset of the radio fall-off is seen. These four requirements can be converted into constraints on the fundamental afterglow parameters $\mathscr{E}$, ε_i, ε_B, and n (external medium density). For the anomalous afterglows we find that reasonable values of these parameters satisfy the observational requirements.

Note: More details on the features of the scenarios described here and the calculations behind the results presented can be found at *astro-ph/0308273*.

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[5] If the energy of the incoming ejecta is dominant, then the exponents q and e are not independent, but they are so for a negligible energy injection, i.e. $e \lesssim 0$